Lambda Calculus $\xrightarrow{\tau}$ CL

that is not just of theoretical interest, but of eminently practical interest to *implement* a functional language on a machine. See, for example:

1. D. A. Turner, "A New Implementation technique for Applicative Languages," Software Practice and Experience, 9, 31-49, 1979.
2. Simon Peyton Jones, "The Implementation of Functional Programming Languages," Prentice-Hall, 1987.

So, what is CL like? In Maude, CL has a very simple definition as ~~[illegible]~~ *system module* [since CL is deterministic, it could also be specified as a *functional module*, replacing the keywords *mod* by *fmod*, and *rl* by *eq*] of the form:

```
mod CL is
sort CL .
ops S K I : CL .  *** basic combinators
op __ : CL CL -> CL  *** application
vars x y z : CL .
 rl [K-red] : (K x) y => x .
 rl [S-red] : ((S x) y) z => (x z)(y z).
 rl [I-red] : I x => x .
 endm
```

The meaning of the K combinator is that $K x$ is the constant function that returns x when applied to any argument y.

The meaning of S is that of a generalized application: x is applied to y and z, and then $(x\ z)$ is applied to $(y\ z)$.

The meaning of I is the identity function.

Exercise. To get a feeling for the translation τ, consider the following instance of the translation:

$$(\star)\quad \tau(\lambda x.(\lambda y.(y\ x))) = \underbrace{(S(K(SI))((S(KK))I)}_{A=}\ ,$$

where, as you can see, names and λ-quantifiers have disappeared! [for τ you can consult several sources such as the Combinatory Logic Wikipedia page, the above two references for Turner and Peyton Jones, and Hindley and Seldin book, "λ-Calculus and Combinators", Cambridge University Press].

Do the following:

1. Specify CL in Maude as suggested in the specification in page 8
2. Apply by hand rules K-red, S-red and I-red to the CL expression $(A\ x)\ y$, where A abbreviates the righthand side of ($\star$) above to check that τ is correct, since it gives the same result as the β-reduction of $(\lambda x.(\lambda y.(y\ x))\ x)\ y$.
3. Compute $(A\ x)\ y$ automatically by the rewrite command in Maude.